Study on waveform inversion method for elastic parameters of stratified medium and its applications

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Abstract: Waveform inversion method was used to reveal the physical and mechanical properties of underground medium, which comprehensively utilized the amplitude, travel time and phase of wave field. A typical stratified medium model in geotechnical engineering investigation was established for grasping the intermediate medium properties. In forward calculation, response waveforms were achieved by finite-difference time domain (FDTD) method and in inversion analysis, the Least Squares method was adopted. From the response waveforms, it is found that the higher the P-wave velocity of intermediate layer is, the smaller the amplitude of response waveform is; while the thicker the intermediate layer is, the more waveform phase is deferred. The feasibility and accuracy of waveform inversion method was verified. Furthermore, this method was used in the detection of grouting quality of immersed tube tunnel. Based on the waveform data collected on site, the P-wave velocity and thickness of grouting layer were revealed by inverse calculation and the grouting quality was effectively evaluated.

Introduction

The study of revealing internal structure of underground medium by varies of detection method has been one of the important and difficult subjects in geotechnical engineering field. And the waveform inversion method is an effective way to grasp the physical and mechanical properties of underground medium (Torantola, 1984). Generally, the waveform inversion method includes two categories. One is based on kinematics method. It mainly utilizes various kinematical parameters (velocity, distance, time and so on), such as the relationship between time and space (t~x), to analyzing the inversed medium. For example, the travel time inversion on essence of ray tracing is widely used in tomography and seismic imaging (Ecker et al., 2000; Song et al., 2003). The kinematics method has advantages of little computation and high precision, but it is difficult to grasp most of elastic parameters. The other one is based on dynamics equations method. It is available for various elastic parameters of underground medium. This method includes dispersion inversion in frequency domain and waveform inversion in time domain (Gauthier et al., 1986; Mora, 1987; Pratt, 1998; Geller and Hara, 1993). However, as dynamics equations are highly uncertain, it will lead to various inversion results if different initial models are input. Because of this dependency, the selection of initial model may come to be a rather troublesome problem. Moreover, the iterative equation is used in dynamics equations method, and rapid calculation for iterative matrix is another important problem.

Similar to the applications of waveform inversion method in geophysics and seismology, in geotechnical engineering investigation, dynamite or hammer striking are also wildly used as excitation source and usually the target structures are simplified stratified medium model. The unknown parameters of medium people concerned are thickness, density, P wave velocity and S wave velocity etc, but many parameters in initial medium model are known in many cases and the number of unknown parameters in inverse calculation is less. So, the applications of waveform inversion method are mainly concentrated on the tomography inversion of underground structure (Song et al., 2010; Xu, 2007). But in geotechnical engineering investigation, the model size is relatively very small and the strict accuracy requirements are strict.

In this work, the waveform inversion method is introduced in detail. The finite difference time domain method (FDTD) is use in solution of elastic wave equations (forward calculation), while the Least Square method is adopted in inverse calculation. A numerical stratified medium model with three layers is established as an analytical case study, by which parameter sensitivity analysis and inverse algorithm verification are carried out. Then the waveform inversion method for stratified medium is used in the site detection for grouting effect in immersed tube tunnel. As the waveform data acquired from grouting site, pretreatments including filtering, phase adjustment, normalization on waveforms are applied. Finally, the unknown parameters of intermediate layer under the tube tunnel is obtained, as a result, the grouting quality is effectively evaluated.

Waveform inversion method

Based on the Least Squares method, the method of waveform inversion analysis for single seismic trace is detailed in this section.

Waveform inversion analysis

Assumed that a medium model contains n unknown parameters (such as density, P-wave velocity and thickness etc.) which are written as parameter vector $\boldsymbol{x}$:

$$\boldsymbol{x}=(x_1, x_2,..., x_n)^T,$$

where x_i is the unknown parameter and T denotes the transpose.

Then by a certain excitation, the response waveform $\boldsymbol{f}$ is recorded from certain seismic trace and it is also considered as forward vector $\boldsymbol{f}$. We can get

$$\boldsymbol{f}=F(\boldsymbol{x}), \tag{1}$$

Here, F is the wave equation operator. Eq. 1 is the statement of forward calculation and worked out by finite difference time domain method (FDTD) based on elastic wave equations in this paper.

At the beginning of the inversion work, we choose an initial model $\boldsymbol{x}_0$ based on some necessary prior estimation. Then the corresponding forward vector $\boldsymbol{f}_0$ is worked out by forward calculation. Ignored higher order terms, Eq. 1 can be approximated by the First Taylor series expansion at the initial model $\boldsymbol{x}_0$ as follow:

$$\boldsymbol{f} = \boldsymbol{f}_0 + \sum_{i=1}^{n} \frac{\partial \boldsymbol{f}_0}{\partial x_i} \Delta x_i \,, \qquad \boldsymbol{\Delta x} = (\boldsymbol{\Delta} x_1,\ \boldsymbol{\Delta} x_2, . \ . \ ., \boldsymbol{\Delta} x_n)^T, \tag{2}$$

where the partial derivative $\partial \boldsymbol{f} \,/\, \partial x_i$ is parameter sensitivity, $\boldsymbol{\Delta x}$ is the correct vector for initial model.

Meanwhile, in actual investigation, the waveform of vibration data at corresponding point are received and recorded as observation vector $\boldsymbol{f}_{\mathbf{obs}}$. The difference between observation vector $\boldsymbol{f}_{\mathbf{obs}}$ and forward vector $\boldsymbol{f}_0$ is defined as an error vector $\boldsymbol{\varepsilon}$.

$$\boldsymbol{\varepsilon} = \boldsymbol{f}_{\mathbf{obs}} - \boldsymbol{f}_0, \tag{3}$$

By the form of 2-norm, the objective function Q is identified as Eq. 4:

$$Q = \left\| \boldsymbol{f}_{\text{obs}} - \boldsymbol{f} \right\|_2 , \tag{4}$$

in which $\| \ \|_2$ is 2-norm operation. Inserted the Eq. 2 and Eq. 3 into Eq. 4, the objective function Q is detailed.

$$Q = \left\| \sum_{i=1}^{n} \frac{\partial \boldsymbol{f}}{\partial x_i} \Delta x_i - \boldsymbol{\varepsilon} \right\|_2 , \tag{5}$$

The destination of inverse work is to find a suitable model with parameter vector $\boldsymbol{x}$ which make the forward vector close to observation vector as possible. In other words, the objective function is made close to zero or reaches the minimum. According to the principle of Least Square Method, when objective function Q reaches the minimum, correction vector $\boldsymbol{\Delta x}$ is the stagnation point of Q and can be solved by the following scheme:

$$\frac{\partial Q}{\partial \Delta x_i} = 0 \Rightarrow \frac{\partial \boldsymbol{f}}{\partial x_i} \cdot \Big(\sum_{i=1}^{n} \frac{\partial \boldsymbol{f}}{\partial x_i} \Delta x_i - \boldsymbol{\varepsilon}\Big) = 0 \Rightarrow \frac{\partial \boldsymbol{f}}{\partial x_i} \cdot \Big(\sum_{i=1}^{n} \frac{\partial \boldsymbol{f}}{\partial x_i} \Delta x_i\Big) = \frac{\partial \boldsymbol{f}}{\partial x_i} \cdot \boldsymbol{\varepsilon} \,, \tag{6}$$

where ‘$|\cdot|$’ means dot product. Equation (6) is a system of linear equations and rewritten in matrix form as follows:

$$\boldsymbol{A \Delta x} = \boldsymbol{B} \,, \tag{7}$$

where $\boldsymbol{A}$ is the *Jacobin* matrix with size of $n \times n$, and $\boldsymbol{B}$ is a coefficient vector. The elements $A_{i,j}$ in *Jacobin* matrix and B_i in coefficient vector are obtained by follows:

$$A_{i,j} = \frac{\partial \boldsymbol{f}}{\partial x_i} \cdot \frac{\partial \boldsymbol{f}}{\partial x_j} \,, \quad B_i = \frac{\partial \boldsymbol{f}}{\partial x_i} \cdot \boldsymbol{\varepsilon} \quad (i = 1, 2, 3, \cdots, n), \quad (j = 1, 2, 3, \cdots, n)$$

Because it is unable or difficult to get the explicit expression of forward vector $\boldsymbol{f}$ in most of cases, the coefficients in Eq. 7 are calculated by numerical approximation method. Here, we apply a slight disturbance δx_i on a certain parameters x_i of medium model, the disturbance vector can be written as below.

$$\boldsymbol{\delta x}_i = (0, 0, ..., \delta x_i \, ..., 0)^T, \quad (i=1, 2, ..., n)$$

where δx_i is generally taken as 5% ~ 10% of initial value of certain parameters x_i. Then the Derivative Quotient is replaced by Differential Quotient.

$$\frac{\partial \boldsymbol{f}}{\partial x_i} = \frac{F(\boldsymbol{x} + \boldsymbol{\delta x_i}) - F(\boldsymbol{x})}{\delta x_i}. \tag{8}$$

Obviously, the *Jacobin* matrix $\boldsymbol{A}$ is a symmetric positive definite matrix. The Eq. 7 will be solved by:

$$\boldsymbol{\Delta x} = \boldsymbol{A}^{-1}\boldsymbol{B}, \tag{9}$$

and the parameter vector $\boldsymbol{x}$ is modified by correct vector $\boldsymbol{\Delta x}$:

$$\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{\Delta x}. \tag{10}$$

Repeat above steps until iterative terminal condition is satisfied. The parameter vector $\boldsymbol{x}$ at the last statement is output and regarded as results of geotechnical structures.

Iterative terminal condition

During inverse calculation, the error vector is closer to zero and the parameter vector $\boldsymbol{x}$ are close to a series of constants, so two terminal conditions are adopted here.

The one is related to error vector $\boldsymbol{\varepsilon}$, in which, the 2-norm of error vector $\boldsymbol{\varepsilon}$ is less than a preset small value η_1.

$$\|\boldsymbol{\varepsilon}\|_2 \le \eta_1, \tag{11}$$

and η_1 is equal to 0.01 here to meet the requirements of precision and accuracy.

The other one is related to correction vector $\boldsymbol{\Delta x}$, in which, the ratio of correction vector $\boldsymbol{\Delta x}$ and medium model $\boldsymbol{x}$ is less than a preset small value.

$$\sum_{i=1}^{n} |\Delta x_i| / \sum_{i=1}^{n} x_i \le \eta_2, \tag{12}$$

and η_2 is equal to 0.01 here. Moreover, in order to avoid the influence of probable numerical oscillations, only if the termination condition is satisfied over three times, the calculation would be break out.

Inversion step

From the description of waveform inversion analysis in section 2.1 and 2.2, the process of inverse calculation can be divided into the following steps:

A. According to the actual conditions in site and the material data known in advance, an initial medium model with parameter vector $\boldsymbol{x}_0$ is established;

B. Improved the recorded vibrations waveforms by some pretreatment means, and considered as observation vector $\boldsymbol{f}_{\mathbf{obs}}$;

C. Carry out forward calculation and get the forward vector $\boldsymbol{f}$.

D. Calculate the error vector $\boldsymbol{\varepsilon}$, as well as its 2-norm value;

E. Get the correct vector $\boldsymbol{\Delta x}$ by Eq. 5 ~ Eq. 9 and the ratio of $\boldsymbol{\Delta x}$ and corresponding parameter vector $\boldsymbol{x}$ by Eq. 12.

F. If the terminal condition is not satisfied, initial parameter vector $\boldsymbol{x}_0$ should be modified by Eq. 10 and repeat steps C~ E.

G. If the terminal condition is satisfied, the inverse calculation breaks out and the parameter vector $\boldsymbol{x}$ is output and regarded as last results.

Case Study

In geotechnical engineering investigation, stratified medium model is wildly used. And lots of cases are related to weak layer, such as the detection of hollowing at the back of tunnel lining, the survey of weak layer underneath strengthening foundation and the evaluation of grouting or sand filling quality in immersed tube tunnel, etc. So, in the case study, we establish a stratified model with three layers for theoretical analysis and numerical verification in this section.

Model

As shown in Fig. 1, a stratified model with three layers is established. It is assumed that the material of first layer is concrete while the third layer is a bearing stratum. The intermediate layer is assumed as defect layer. Their physical property parameters were listed in Table. 1, where ρ and v_p are the density and P-wave velocity respectively, h is the thickness and μ is the Poisson's ratio. The target parameters to be revealed are only the P-wave velocity and thickness of defect layer.

Table.1 Material parameters in stratified model

Layer	Materials	ρ (kg/m^3)	v_p (m/s)	h (m)	μ
Material-1	Concrete	2400	3600	1.4	0.2
Material-2	Defect layer	1800	v_p	h	0.35
Material-3	Bearing stratum	2000	2800	3.2	0.3

The Perfect Matched Layer (PML) absorbing boundary (Virieux, 1986) is used on the left, right and bottom boundary of numerical model while the boundary condition on top is set as free. A seismic detector is set at upper boundary and 50 cm away from excitation node and the time-history response waveform is recorded as forward vector $\boldsymbol{f}$.

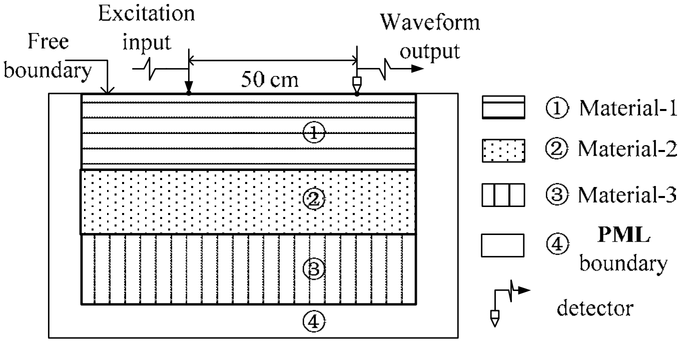


Fig.1 Stratified model with three layers.

Sensitivity analysis of parameters

In order to clarify the sensitivities of target parameters to the response waveform, nine conditions (listed in Table. 2) with different values of v_p and h are calculated respectively. The forward calculation results under each condition are shown in Fig. 2. According to the comparison on waveforms between each other, the sensitivities of target parameters to forward vector $\boldsymbol{f}$ are clarified.

Table.2 Analytical Conditions in forward calculation

Conditions	v_p **=1700 m/s**	v_p **=2400 m/s**	v_p **=3000 m/s**
h=0.1 m	A	B	C
h=0.4 m	D	E	F
h=0.7 m	G	H	I

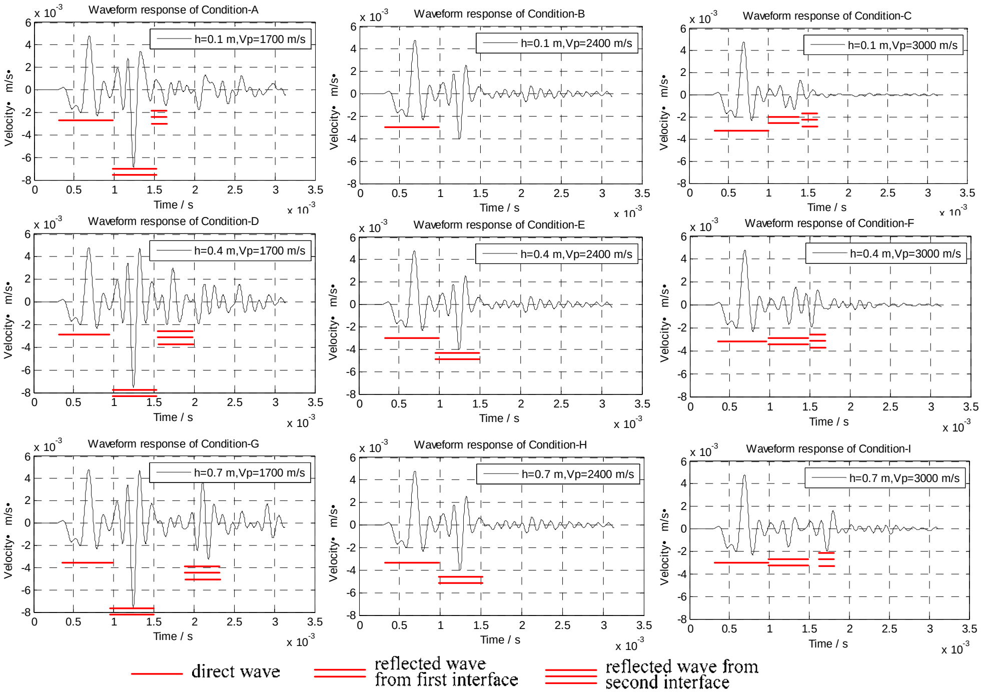


Fig.2 Forward calculation results under each analytical condition.

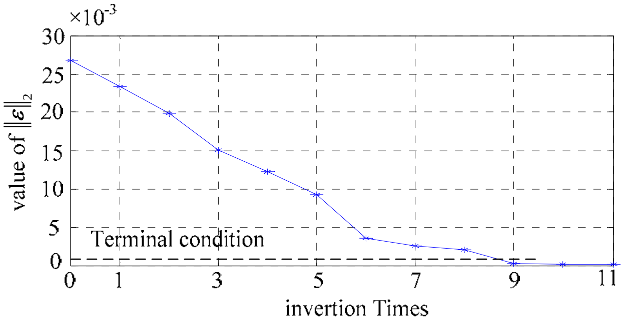


Fig. 3 Value of 2-norm of error vector decreased during the iterations.

From Fig. 2, waveform shapes within the first 1 ms in the nine charts are exactly the same, so this part of waveform can be regarded as direct wave.

From 1 ms to 1.5 ms, the dominant component of waveform is the reflected wave from first interface. When the thickness h of intermediate layer is fixed, the waveform amplitude gets smaller along with the increase of P-wave velocity v_p (comparisons between row direction). It can be explained that when v_p increases, the reflection coefficient on the interface between the first layer and intermediate layer relatively reduces. Therefore, more energy transfers into the intermediate layer and the bottom layer.

From 1.5 ms to 2.5 ms, the dominant component of waveform is the reflected wave from second interface. When P-wave velocity v_{p} is fixed, the waveform shapes appear obvious phase lag by the increasing of thickness h of intermediate layer (comparisons between column direction). Because the reflected wave path is increased, the duration of reflected wave arrived at the detector lasts longer as a result. It is must be mentioned that, in the conditions of second column, since the material parameters of defect layer are close to the bearing stratum, so the reflected wave from second interface is too weak to be clarified in waveforms.

Inversion analysis

Here, the waveform inversion method is applied into the inversion of the two parameters. At first the target parameters were preset as h = 0.6 m and v_{p} = 2500 m/s and the responded waveform was considered as observation vector $\boldsymbol{f}_{\mathbf{obs}}$. Then the initial values of the parameters are reset as h = 0.4 m and v_{p} = 1700 m/s.

As the waveform inverse calculation was carried out, the inversed results was that the thickness h = 0.64 m and the P-wave velocity v_{p} = 2501 m/s. Compared with the preset values, the relative error are 6.7% and 0% respectively. The value of 2-norm of error vector $\boldsymbol{\varepsilon}$ was decreased during the iterations, as shown in Fig. 4. It is verified that the waveform inversion method for stratified medium is available for revealing the elastic physical parameters of intermediate layer.

Applications on grouting quality evaluation

General

The project of immersed tube tunnel under Haihe River is located in Yujiabao central business district in Tianjin Binhai New Area. The immersed tube tunnel is overall 255 m in designed length and the thickness of its tube floor is 1.4 m, where the strength grade of concrete is C40. In order to satisfy seismic resistance requirements, the mortar foundation is adopted instead of traditional sand foundation.

Before grouting construction, an intermediate space would be preserved between undersurface of tunnel floor and the flattened gravel trench and it was full of saturated soil usually. And After grouting construction, the intermediate space was full filled by mortar. When the mortar layer solidified, the immersed tube tunnel would lay on the mortar base. So the high quality level of mortar foundation is important for reducing the uneven settlement of tunnel and ensuring the safety in daily operations during the service life.

Waveform acquisition in site

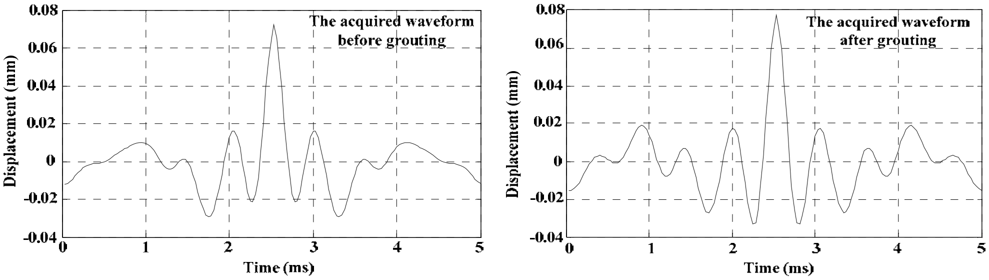
The waveform acquisition work in site was taken on three days after the grouting construction.

(a) Measurement line (b) Hammer (c) Receiver (d) Seismograph

Fig. 4 Equipments for acquisition work in field.

The sketch of measurement line in site was shown in Fig. 4-a. In waveform acquisition work, a round head hammer with a weight of about 1.5 pounds was used as hypocenter (shown in Fig. 4-b). The receiver (shown in Fig. 4-c) which was 0.5 m away form the strike point will detect the vibration signals. A small seismograph (shown in Fig. 4-d) accepted the signals from receiver, then displayed and recorded them on a notebook PC in real time.

Because the various noise vibrations in work field came from grouting equipment and ship traffic and most of them were in low frequencies band, the acquired waveform should be filtered by band-pass whose frequencies were in band of 300 Hz ~ 3000 Hz. Moreover it is inevitable that the distance between strike point and receivers could not be kept at 50 cm perfectly in actual site, so the collected waveform possessed more or less initial phase difference. In order to eliminate its influence, the initial phase of the acquired waveform was artificially set zero. Besides, as the hammer was manually stroked by workers, the source strength in each excitation was uncontrollable and imprecise. As a result, the amplitudes of acquired waveforms were not consistent with each other, so it is necessary to take normalization on waveforms. In this work, energy normalization is adopted. Finally after the above pretreatment, the acquired waveform was improved, as shown in Fig. 5.



(a) Before grouting (b) After grouting

Fig. 5 Acquired waveforms after the pretreatments.

Initial model

According to the description on grouting construction, a stratified medium model with three layers was established. Similar to the theoretical model in section 3.2, the first layer was concrete, the intermediate layer was saturated soil before grouting and would be replaced by mortar after grouting, and the third layer was gravel and sludge. In the inverse calculation, some assumptions were made as following:

1. All of the materials except the saturated soil in the model were assumed isotropic elastic.

2. When the intermediate layer was full of saturated soil before grouting, its S-wave velocity v_s and P-wave velocity v_p can be estimated by Eq. 13 (Xia, et. al, 2004).

$$v_\mathrm{s} = \sqrt{G/\rho}\ , \quad v_\mathrm{P} = \sqrt{\frac{(\lambda + 2G) + E_w / n}{\rho}}\ , \tag{13}$$

Where E_w is bulk modulus of water and equal to 2100 MPa; n is the pore ratio of saturated soil and taken as 0.4 in this work; G is the shear modulus of saturated soil and assumed as 160 MPa; λ is the lame constant and taken as 510 MPa.

3. Based on the actual conditions in grouting construction of immersed tube tunnel, the thickness of intermediate layer was somewhere between 0.4 and 0.6.

Therefore, if the value of thickness was beyond its extent during the iterative calculation, it required corresponding correction.

So the material parameters of each layer are listed in Table. 4.

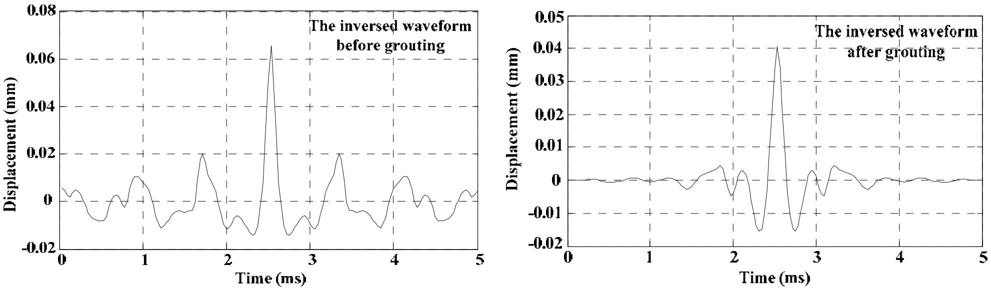
Table.4 Material parameters of each layer

Layers	**Material Name**	**ρ (kg/m^3)**	**v_p (m/s)**	**h (m)**	**μ**
1	Concrete	2400	3600	1.4	0.18
2-a	Saturated Soil	1800	v_p	h	0.35
2-b	Motor	2100	v_p	h	0.3
3	Gravel & Sludge	1900	2800	3.2	0.3

Waveform inversion before/after grouting

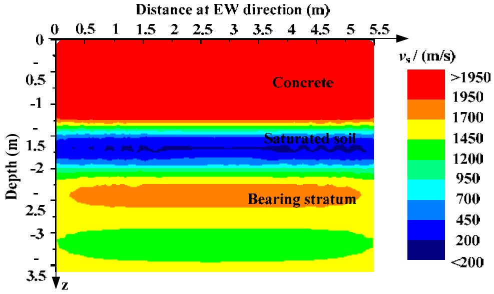
Before grouting, in line with the design and construction schemes, the intermediate layer under the immersed tube tunnel was full of saturated soil with an approximate thickness of 0.5 m. At the beginning of waveform inversion calculation, the initial parameters of intermediate layer were preset as h = 0.5 m, v_p = 1600 m/s. The after plenty times of iteration, the results were reached and turned out to be h = 0.58 m and v_p = 1753 m/s. Then by using Eq. 13, the S-wave velocity at this state was determined as v_s = 221 m/s. The final inversed waveform before grouting was shown in Fig. 6 (a).

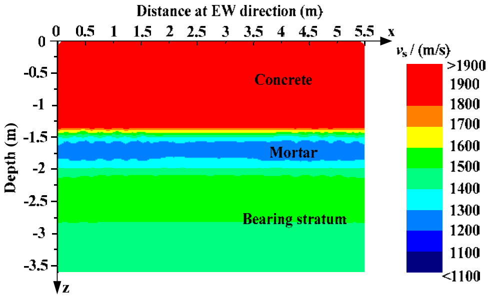
After grouting construction, the intermediate soil layer was replaced by a mortar base with a certain degree of solidification. So, before inversion calculation, the initial parameters of intermediate layer were reset as h = 0.5 m and v_p = 2000 m/s. Thereafter, calculation was performed with the results of h = 0.6 m and v_p = 2063 m/s. According to the elastic assumptions, the S-wave velocity was deduced as v_s = 1290 m/s. The final inversed waveform after grouting was shown in Fig. 6 (b).



(a) Before grouting (b) After grouting

Fig. 6 Inversed waveform before/after grouting construction.





(a) Before grouting (b) After grouting

Fig. 7 S-wave velocity profiles under a measure line before/after grouting.

The inversion work shown above had revealed the S-wave velocity of intermediate layer before and after grouting construction respectively. The S-wave velocity has been highly enhanced from 221 to 1290 m/s, which mean that the grouting quality at the detection point was well. Similarly, by enough times of inversion work on each detection points of a measure line, the approximate profiles of S-wave velocity under the measure line before and after grouting are achieved, as shown in Fig. 7. Compared with S-wave velocity profiles, it was concluded that the grouting quality under the immersed tube tunnel was good.

Conclusions

From the numerical results of stratified model, it is concluded that the response waveform amplitude is reduced with the increase of higher P-wave velocity v_p of intermediate layer; and the thicker the intermediate layer is, the more the reflected waveform phase is deferred.

The results of inversion work in the theoretical stratified model verify that the waveform inversion method is capable of revealing the elastic physical parameters of intermediate layer.

Through enough times of inversion work for each detection point of a measure line, the approximate profiles were achieved. By comparing the parameters of intermediate layer before and after grouting, it reveals the high grouting quality under the immersed tube tunnel. The waveform inversion method of stratified medium has been proved to be feasible for evaluating the grouting quality under immersed tube tunnel.

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